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NITRATE LEACHING FROM A POTATO FIELD USING FUZZY INFERENCE SYSTEM COMBINED WITH GENETIC ALGORITHM

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ABSTRACT: The conventional application of nitrogen fertilizers via irrigation is likely to be responsible for the increased nitrate concentration in groundwater of areas dominated by irrigated agriculture. This requires appropriate water and nutrient management to minimize groundwater pollution and to maximize nutrient use efficiency. To fulfill these requirements, drip fertigation is an important alternative. This study deals with fuzzy modeling of nitrate leaching from a potato field under a drip fertigation system. In the first part of the study, a two-dimensional model (HYDRUS-2D) was used to simulate nitrate leaching from a sandy soil with varying emitter discharge rates and various amounts of fertilizer. The results from the modeling were used to train and validate a Mamdani fuzzy inference system (MFIS) in order to estimate nitrate leaching. The centers of triangular membership functions in MFIS were tuned by Genetic Algorithm. The correlation coefficient, normalized root mean square error and relative mean absolute error percentage between the data obtained by HYDRUS-2D and the estimated values using MFIS model were 0.986, 0.086 and 2.38 respectively. It appears that MFIS can predict nitrate leaching from the field accurately. The proposed methodology can be used to reduce the effect of uncertainties in relation to field data.

Key words: Drip fertigation; Fuzzy logic; Genetic algorithm; HYDRUS-2D; Nitrate leaching

INTRODUCTION

The quality of soils, ground and surface waters is vulnerable in climatic regions where irrigation agriculture dominates. Here regular excessive application of nitrogen fertilizers with irrigation water is likely to be responsible for the increase in nitrate concentrations of the groundwater resource. Therefore, alternative irrigation water and soil management practices are needed to maximize the application efficiency of water and fertilizer thereby minimizing leaching of nitrogen out of the root zone into the groundwater (Bar-Yosef, 1999). Transport processes of water and nutrients in relation to fertigation systems might be complex. Conducting field experiments with varying emitter discharge rates and fertilizer to investigate water and nutrient distribution for evolving appropriate design and management option is both costly and time consuming. A properly calibrated and validated water flow and solute transport model can reduce time and cost required for studying the water and nutrient dynamics under drip irrigation systems (Antonopoulos, 2001).

Many researchers have reported that the HYDRUS-2D package (Simunek et al, 1999) is a convenient tool for modeling and simulation of nitrogen under drip irrigated conditions (Cote et al., 2003; Gardenas et al., 2005; Ajdary et al., 2007; Dultra and Munoz ., 2010). HYDRUS-2D needs a high number of input parameters and the measurement of many of these is often time-consuming and difficult and also the calibration of the parameters is difficult and time consuming. Also running these models for a long time span such as the entire growing season is time consuming. Therefore, there is a necessity to use simpler, less time consuming models. One alternative is fuzzy rule based models which they need a small amount of input parameters since these models are knowledge-based and expert can apply their knowledge in them. Also the computational time using fuzzy models is much smaller. Fuzzy logic was first introduced by Zadeh (1965) and has been successfully utilized for water flow and solute transport processes (Dou et al., 1997; Freissinet et al., 1998; Schulz and Huwe., 1999; Kettle et al., 2002; Senevirathna et al., 2006; Verma et al., 2007).

Dou et al. (1999) used a fuzzy rule based approach to describe solute transport in the unsaturated zone through a soil column using Mamdani fuzzy inference system (MFIS, Mamdani and Assilian, 1975). They used simulation results of the SWMS-2D model to obtain a data set for fuzzy rule derivation and to verify fuzzy rules and reported a good correspondence between the outputs from the two models. Also they found that outputs from the fuzzy rule based model were in accordance with the results from the solute transport experiment.

Genetic algorithm (GA) is inspired by Darwin's theory about evolution, which strengthens survival ability by the processes of reproduction, crossover and mutation of genes. Genetic algorithms (GAs) have been shown to be more effective compared to classical optimization methods (Holland, 1975; Goldberg, 1989).GAs have proven to be effective and robust tools for finding the solutions to optimization problems. GAs have been used in optimization of nonlinear problems in a wide variety of fields, including water resources and environmental engineering due to their robustness and general applicability to various problems. For example, GAs have been used in optimization of water distribution systems (Gupta et al., 1999), and in relation to solute transport in the soil (Massoudieh, et al, 2008).

In this study, HYDRUS-2D was used to obtain training and test sets of MFIS. According to our knowledge no attempt has been made yet to use fuzzy logic in the prediction of solute transport on a field scale.

The objective of this study is to develop a MFIS combined with GA for prediction of nitrate leaching from a potato field under drip fertigation.

MATERIALS AND METHODS

Experimental details and measurements

The experiment was conducted in an agricultural experimental station in the city of Jiroft in the Kerman Province located in the southern part of Iran (latitudes 26 N and 29N longitudes 56E and 59E) during 2009-2010. The soil textural class was a sandy soil according to the USDA classification system. The climate is categorized as semi-arid with a mean annual temperature of 27.8 °C and a mean annual rainfall of 175 mm. Before planting of the potatoes the field was heavily irrigated twice to leach excess salts out of the root zone. Irrigation water was applied at a rate of 1 L h^{-1} through drip emitters placed at the soil surface parallel to and within the crop row. Distance between each emitter was 20 cm and distance between each potato row was 60 cm. Potassium nitrate (KNO_3) was used as fertilizer in the fertigation system and was applied through the irrigation water. Fertigation was started immediately after the emergency of the potato plants**.** A total of 600 mm irrigation water and a total of 200 kg N ha^{-1} were applied through the fertigation system during the entire growing season. Application of N fertilizer was performed six times during the growing season in regularly amounts. We used an irrigation and fertigation schedule typically practiced by the farmers cultivating potato in the region.

To obtain a data set for calibration and validation of the modeling, soil samples were collected from different depths (0–0.2, 0.2–0.4, 0.4–0.6 m) at a horizontal distance of 0, 0.15 and 0.30 m from the emitter perpendicular to the row direction using a tube auger to determine spatial and temporal distribution of water and nitrate during the growing season. The samples were collected immediately before the first fertigation and after that before each fertigation event. In addition, samples were taken after selected irrigation events through the growing season. In the laboratory, soil samples were analyzed to determine the gravimetric moisture content. The nitrate concentration in the soil was measured using the spectrophotometer method (Page et al., 1982).

HYDRUS-2D

To model nitrate leaching from the potato field under drip fertigation the computer simulation model, HYDRUS-2D (Simunek et al., 1999) was used. This software package can simulate the transient two-dimensional movement of water and nutrients in soils. In HYDRUS-2D (Simunek et al., 1999) solute transport is described by

$$\frac{\partial \bar{}c}{\partial t} = \frac{\partial}{\partial x_i}(\bar{}D_{ij}\frac{\partial c}{\partial x_i}) - \frac{\partial q_i c}{\partial x_i} - NU(c,r,z,t) \qquad (1)$$

where Ɵ is the volumetric soil water content (L^3 L^{-3}), *q* is volumetric flux density (L T^{-1}), *t* is time (T), the subscripts *i* and *j* denote either *r* or *z*, and *c* denotes the nitrate concentration in soil solution (M L^{-3}), D_{ij} is the dispersion coefficient (L^2 T^{-1}), and the *NU* term defines the local passive nitrate uptake (M L^{-3} T^{-1}) by plant roots, which is a function of time and space. Initial condition for water was given as initial water content in different soil layers within the flow domain as observed. A 60 × 60 cm^2 domain was used in the modeling with a no flux boundary condition at the sides. For the lower boundary condition, free drainage was used since the water table was situated far below the domain of interest. To take into account the emitter discharge during irrigation, a flux type boundary condition was used. Nitrate fertigation was applied with irrigation water and a third-type Cauchy boundary condition was used to describe the concentration flux along flux variable at the top boundary. The model was calibrated with respect to the hydraulic conductivity and dispersivity values of water and nitrate at various sampling points. Calibrated parameters were selected when the mean correlation coefficient (R) between predicted and observed values was higher than 0.951. After calibration, the model was validated to examine its predictability. During calibration runs, simulation period was kept to 267 h, which included two fertigation and 6 irrigation events. For the validation, simulation period was kept to 3000 h equal to the growing period of the potato. For the various hydraulic input parameters required in HYDRUS-2D saturated water content ($Ɵ_s$), residual water content ($Ɵ_r$), empirical factors (α, n) and saturated hydraulic conductivity (K_s) were obtained from the neural network option available in HYDRUS-2D to parameterize the Mualem-van Genuchten (Mualem, 1976; van Genuchten, 1980) analytical model without hysteresis. The l value of van Genuchten-Mualem model was set to 0.5. Values of longitudinal and transverse dispersivity confirmed through calibration were 8 and 0.8 cm, respectively. After calibration and validation, the model was used to predict the nitrate leaching below the root zone. Here emitter discharge rates were varied from 0.5 to 8 L h^{-1} with increments of 0.5 L h^{-1} and the amounts of potassium nitrate were varied from 950 to 2550 kg ha^{-1} with increments of 50 kg ha^{-1} yielding a total of 528 scenarios simulating and evaluating the nitrate leaching out of the root zone of the soil. In the simulation and validation we focused on one emitter only and the reported nitrate leaching was obtained from the same emitter.

Genetic Algorithms

GAs are an exploratory search and optimization procedure based on the principle of natural evolution and population genetics. The basic concepts of GAs were proposed by Holland (1975). Commonly, there are two kinds of GAs: binary GAs (BGAs) and real-valued GAs (RGAs). In this study, a RGA was used. Figure 1 shows the procedure of a typical RGA. The definition of initial parameters of the GA includes number of generations, number of variables, lower and upper bound of variables, crossover and mutation probabilities. The definition of the objective function is the first step to apply GA and the value of objective function for each individual is usually used as a measure of the individual's fitness. In this study, the minimum relative mean absolute error percentage was considered as the main objective. In order to avoid the variable scale, which would influence the criterion, mean absolute error percentage (RMAEP) is usually used as a criterion to show the dimensionless relative deviation (Vicente, 1996).

In a GA, first a population of points (solutions) is generated randomly. The algorithm starts with a set of solutions called population, which is analogous to the chromosomes (individuals) in the natural systems. For every chromosome in the population the fitness is computed. The probability that an individual may survive and reproduce in the next generation is proportional to its performance (fitness value associated with it). As mentioned before, in this study the objective function and fitness function are defined as follows.

$$\text{Objective function=RMAEP=}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{|Y(p_i)-Y(o_i)|}{Y(o_i)}\right)\times 100 \qquad (2)$$

Fitness function=100-R MAPE (3)

where $Y(p_i)$ and $Y(o_i)$ are observed and predicted values of leached nitrate respectively, and n is the number of data samples.

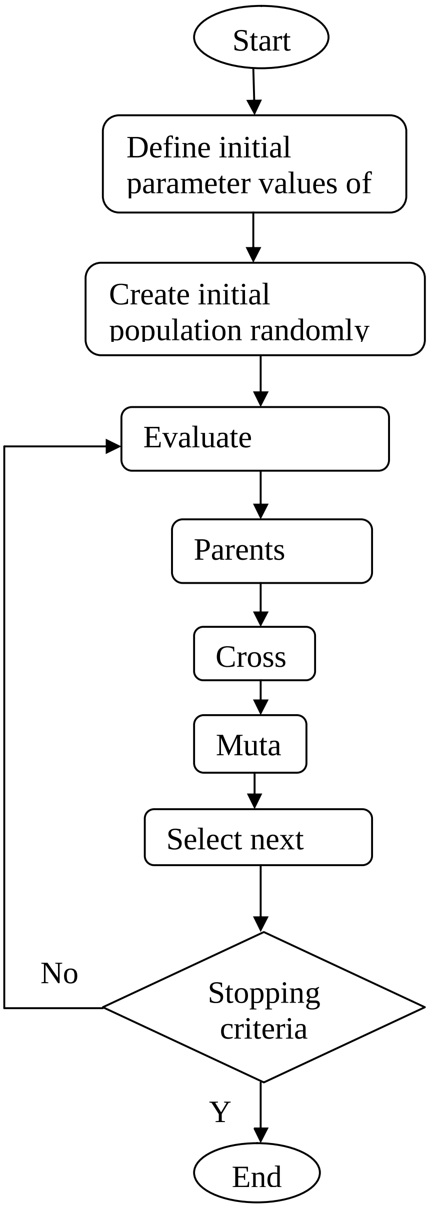

Figure 1. Blocky diagram of a real genetic algorithm

In this study a fitness-proportionate method also called roulette wheel selection (Goldberg, 1989) is utilized to select individuals for reproduction based on their fitness values. After parents selection the genetic operation of crossover is performed on each mated pair with a certain probability, referred to as crossover probability. The common crossover operations can be uniform, single-point, two-points and arithmetic crossover (Michalewicz, 1994). For a RGA, arithmetic crossover is simple and effective. In this study, an arithmetic crossover was selected and designed to the crossover operation. In the next step a mutation operation was applied. Mutation is an operator which is used to maintain the diversity in the population and to allow the algorithm to avoid local optima by preventing the individuals in a population from becoming too similar. Mutation randomly changes some bit in a binary sequence representing an individual according to the mutation probability. In this study, a gaussian mutation was selected and designed to the mutation operation. After the produce of next generation (offspring), stopping criteria was checked and the algorithm is repeated until a specified termination criterion, such as a limit on the maximum number of generation or no obvious change of fitness or preset fitness, is satisfied.

In this study RGA is used for tuning the centers of triangular membership functions in a MFIS. Table 1 shows the best parameters of GA for MFIS model.

Fuzzy inference system (FIS)

The generic structure of a FIS is shown in Figure 2. A FIS is composed by a knowledge-base, that includes the information given by the expert in the form of linguistic fuzzy rules, a fuzzifier, which transforms the crisp inputs into degree of match with linguistic values, an inference system (engine), that uses them together with the knowledge-base to make inference by means of a reasoning method, and a defuzzifier, which transforms the fuzzy results of the inference into a crisp output using a defuzzification method. The knowledge-base is composed of two components: a data-base, which defines the membership functions of the fuzzy sets used in the fuzzy rules and a rule-base comprised of a collection of linguistic rules that are joined by a specific operator (Herrera and Lozano, 2003).

Table 1. Values of GA parameters in MFIS model.

GA paremeters	values
Crossover probability	0.6
Mutation probability	0.01
Number of generations	100
Number of variables	25
Number of iterations	50

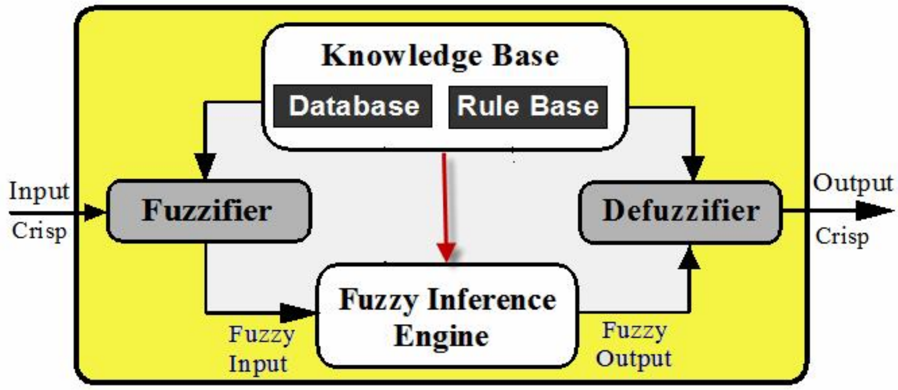

Figure 2. A flow diagram of a fuzzy inference system (FIS).

Fuzzy membership functions may take many forms, but in practical applications simple linear functions such as triangles are preferable. In this study, emitter discharge, fertilizer amount, and nitrate leaching were fuzzified into fuzzy subsets using triangular membership functions. The various number of membership function was tried and finally we found with selection seven membership functions for emitter discharge, nine membership functions for fertilizer amount and nine membership functions for nitrate leaching the error was at least. Centers of triangles for all membership functions are tuned with the RGA.

The rules are expressed in the IF-THEN format. The fuzzy system employs the inference method proposed by Mamdani, in which the rule consequence is defined by fuzzy sets and has the following structure (Mamdani and Assilian, 1975).

IF x is A and y is B THEN *f* is C

(4)

where A, B and C are fuzzy membership functions, x and y are inputs and *f* is the output of the fuzzy inference system.

In this study, fuzzy rules relating the emitter discharge and fertilizer amount to nitrate leaching are inferred from the experimental data. The antecedent part of the rule included a statement on the emitter discharge and fertilizer amount while the consequent part included a statement on nitrate leaching. Table 2 summarizes the fuzzy rules constructed in this study. There are several defuzzification methods, such as the weighted average, maximum membership, average maximum membership, center of gravity, etc. In this study center of gravity was used.

We had two sets of input data: emitter discharge rate and fertilizer amount and one set of output data: nitrate leaching. The data set had a total of 528 data points. This was divided into two smaller sets: a training data set (352 data points) and a testing data set (176 data points). The training data was used to derivate fuzzy rules and optimizing parameters of the MFIS. The ability of the model in prediction of nitrate leaching was evaluated using test data. The model was implemented in the MATLAB software.

Table 2. Rules of Mamdani fuzzy inference system.

		Emitter discharge membership						
		1	2	3	4	5	6	7
Fertilizer amount membership	1	1	3	3	3	3	3	3
	2	1	3	4	4	4	4	4
	3	2	4	4	4	4	4	4
	4	2	4	5	5	5	5	5
	5	2	5	6	6	6	6	6
	6	3	6	6	7	7	7	7
	7	3	6	7	7	8	8	8
	8	3	7	8	8	8	8	8
	9	3	7	8	8	9	9	9

In this table for example (1,1,1) means: If emitter discharge is 1 and fertilizer amount is 1 Then nitrate leaching is 1.

Performance criteria

The correlation coefficient (*R*), relative mean absolute error percentage (RMAEP) and normalized root mean square error (NRMSE) between the measured and the estimated nitrate leaching values were used to evaluate the performance of the MFIS model. The MAPE, R and NRMSE are denoted as belo

w

$$\text{RMAEP}=\left(\frac{1}{n}\sum_{i=1}^{n}\frac{\left|Y(p_i)-Y(o_i)\right|}{Y(o_i)}\right)\times 100 \qquad (5)$$

$$R=\frac{\sum_{i=1}^{n}(Y(p_i)-\overline{Yp})(Y(o_i)-\overline{Yo})}{\sqrt{\sum_{i=1}^{n}(Y(p_i)-\overline{Yp})^2\,(Y(o_i)-\overline{Yo})^2}} \qquad (6)$$

$$\text{NRMSE}=\sqrt{\frac{\sum_{\text{i}}^{\text{n}}(Y(o_i)-Y(p_i))^2}{\sum_{i}^{n}(Y(o_i)-\overline{Yo})^2}} \qquad (7)$$

where $Y(p_i)$ and $Y(o_i)$ are observed and predicted leached nitrate values respectively, Yp and Yo are the means of observed and predicted nitrate leaching values respectively and n is the number of data points.

Results and Discussion

Table 3 shows maximum, minimum, median, mean, variance, and standard deviation (SD) of the data. The emitter discharge rates and various amount of fertilizer (KNO_3) were used as premises and nitrate leached from an emitter was used as the consequence part of the rules in the MFIS model.

Table 3. Statistical characteristics of variables.

	Emitter discharge rate (l h^{-1})	Fertilizer (kg ha^{-1})	Nitrate leaching (mg)
Maximum	8	2550	3161
Minimum	0.5	950	24450
Mean	4.25	1750	15228
Median	4.25	1750	15200
Variance	5.322	227097	24509516
Standard Deviation	2.307	477	4951

Figure 3 shows the average fitness and best-so-far result (best result from first generation to so far) of chromosomes for training data for all generations. The best so far result was 97.88. RMAEP for the training data was 2.12. The R coefficient between MFIS data and HYDRUS-2D data for training data was 0.99 (Figure 4). These results show that MFIS combined with GA can capture relationships between premises part (input data) and consequence part (output data) with good accuracy. GAs have proven to be effective and robust tools for finding the solutions to optimization problems (Davis, 1991).The performance criteria values for testing data are presented in Table 4. The correlation coefficient was 0.986 (Figure 5). This is a high accuracy considering the complex mechanisms of nitrate transport under drip fertigation. The RMAPE and NRMSE for testing data were 0.086 and 2.38 respectively Values of performance criteria show that the MFIS model can be applied for prediction of nitrate leaching in a fertigated crop such as potato with good accuracy.

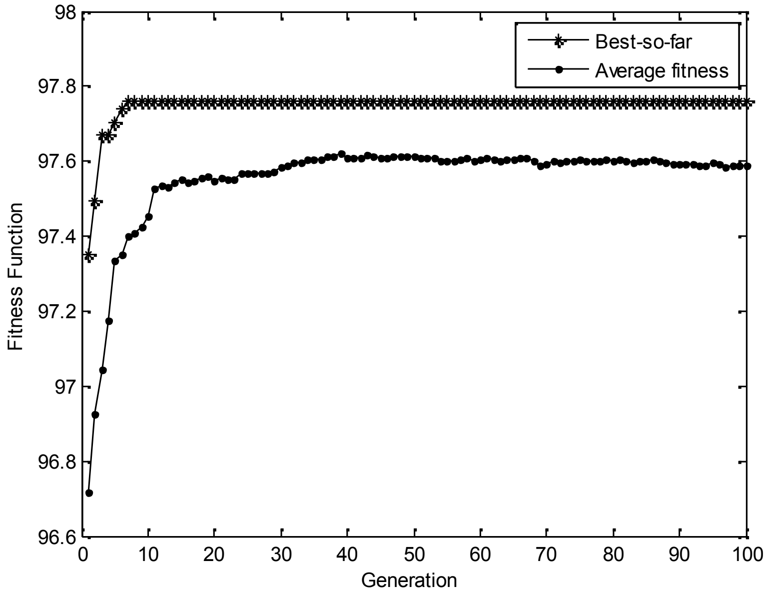

Figure 3. Fitness function versus generation number for MFIS model

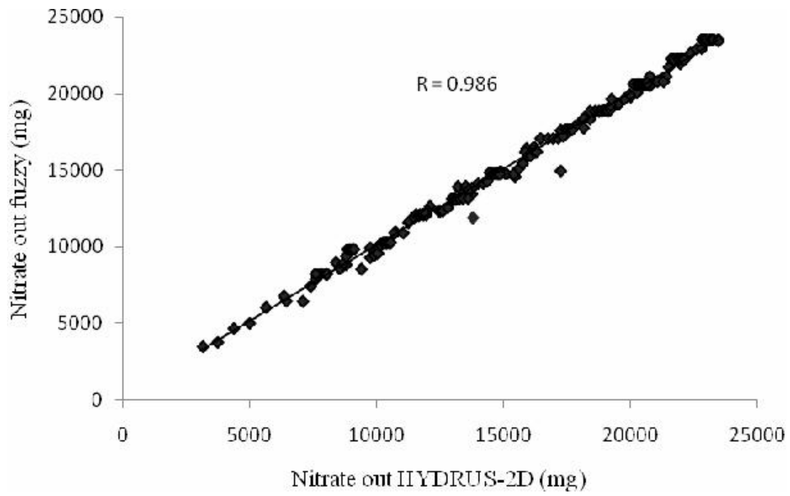

Figure 4. Estimates of cumulative nitrate leaching during entire growing season by the MFIS model using training data versus outputs from HYDRUS-2D

Table 4. Values of performance criteria in MFIS model.

Performance criteria	MFIS
Correlation Coefficient (R)	0.986
Mean Absolute Percentage Error (MAPE)	2.38
Normalized Root Mean Square (NRMSE)	0.086

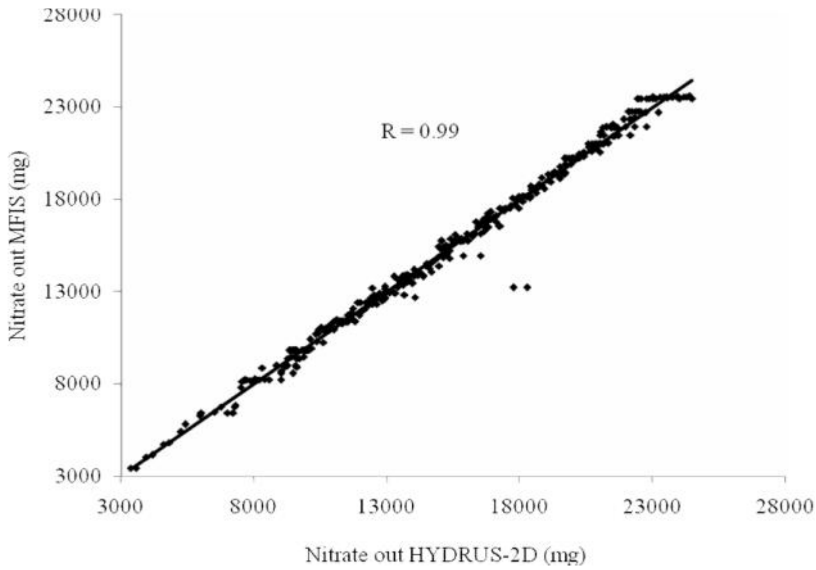

Figure 5 Estimates of cumulative nitrate leaching during entire growing season by the MFIS model using test data versus outputs from HYDRUS-2D.

No other study has investigated solute transport using fuzzy inference system on a field scale. Dou et al (1999) used MFIS to investigate solute transport through a soil column. They reported that accuracy of fuzzy rule base model in prediction solute transportation was good.

By optimizing the rules, the accuracy of the fuzzy model could be improved. One way to optimize the rules is to apply the least squares algorithm proposed by Bardossy and Duckstein (1995) to derive the fuzzy rules. In order to optimize rules in our study we used GA. Another possibility is to use artificial neural nets to improve the assessment of membership functions in the fuzzy rules (Muster et al., 1994).

CONCLUSIONS

In this study, the efficacy of MFIS in combination with GA in estimating nitrate leaching from a potato field from the results of HYDRUS-2D was investigated. In this study the effect of the emitter discharge rates and various amount of fertilizer using a MFIS model predicting nitrate leaching was investigated. The comparison between the results of the MFIS and HYDRUS-2D model showed that the overall accuracy of the MFIS model was high. MFIS model demonstrated a powerful tool to study solute and water transport in the soil and in comparison to the numerical model the calculation time was significantly decreased. The proposed methodology can also be used to reduce the effects of probable errors and uncertainties in field data and laboratory and sampling analyses.

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